STAT 321

Sample Space (S)

"List of possible outcomes for a random experiment where the outcome is unknown"

Event: (A, B, C, ...)

"Subset of a Sample Space"

$A = \{2,4,6\}$

$B = [0, 60]$

Null set: (ϕ)

"An event with no outcome"

ex: E = roll a number > on a die

$E = \phi$

Complement: (NOT)

"The outcomes in the sample space not covered by an event"

$S^c = \phi$

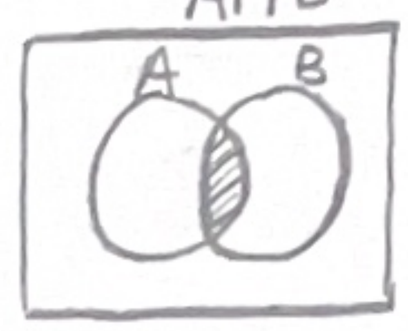
Union: (OR)

$A \cup B$

Intersection: (AND)

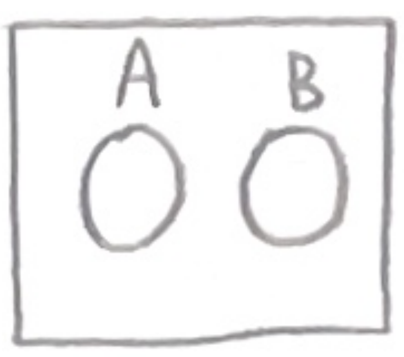
$A \cap B$

Venn Diagram:



Mutually Exclusive:

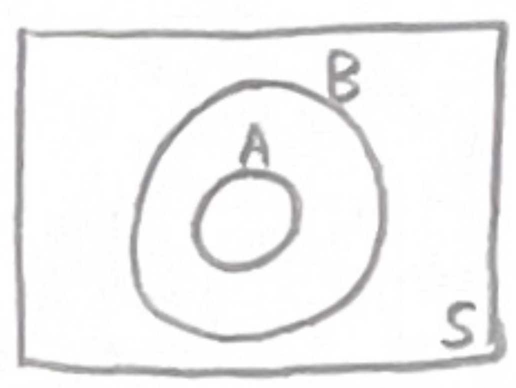
$A \cap B = \phi$



Contained Event:

$A \subset B$ 'or' $B \supset A$

"Little guy eats the big guy"



Axioms of Probability:

① $0 \leq P(E) \leq 1$ (Event; Probability function)

② $P(S) = 1$ (sample space)

③ For a sequence of mutually exclusive events,

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

Probability Rules:

① $P(E^c) = 1 - P(E)$

② If $E \subset F$, $P(E) \leq P(F)$

③ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

④ If all outcomes are equally likely

$$P(E) = \frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S}$$

⑤ **Inclusion - Exclusion formula**:

$P(A_1 \cup A_2 \cup A_3)$

$= P(A_1) + P(A_2) + P(A_3)$

$- P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3)$

$+ P(A_1 \cap A_2 \cap A_3)$

Combinatorial Formula: $(r \leq n)$

"The number of distinct combinations of size r that can be made from n elements"

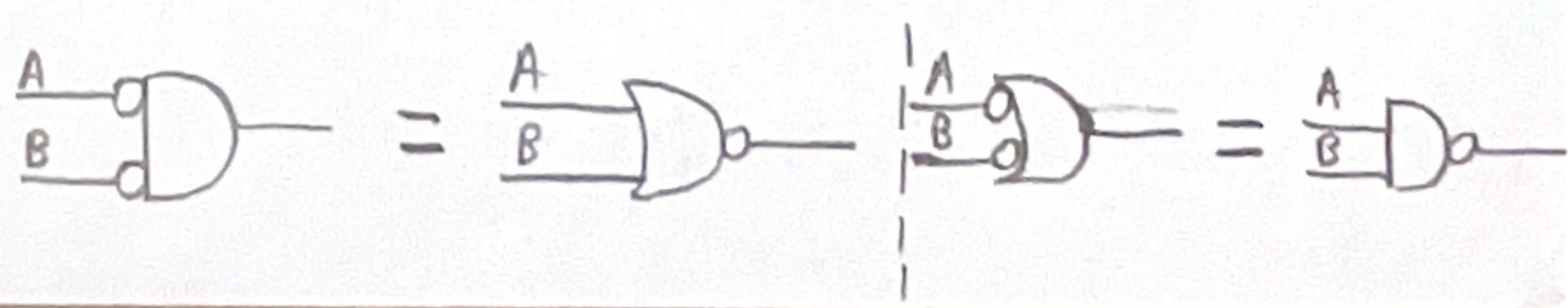
$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(order doesn't matter, just unique sets)

De Morgan's Law

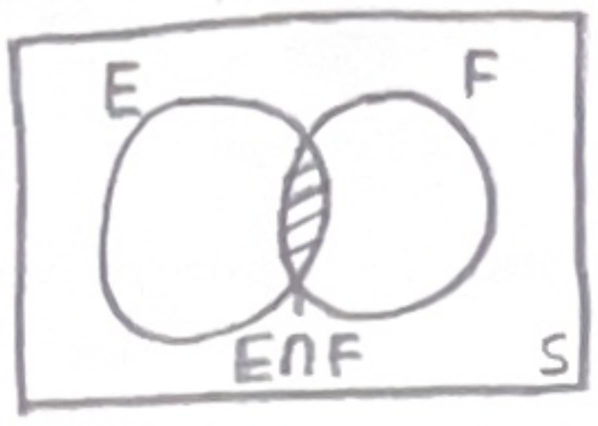
$P(A^c \cap B^c) = P((A \cup B)^c)$

$P(A^c \cup B^c) = P((A \cap B)^c)$



Conditional Probability

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



"The probability that event E occurs given the fact that event F has already occured"

Bayes Thereom:

$$P(E|F) = \frac{P(F|E) \cdot P(E)}{P(F)}$$

Conditional Independence: (For E and F)

- $P(E|F \cap G) = P(E|G)$
- $P(F|E \cap G) = P(F|G)$
- $P(E \cap F|G) = P(E|G) \cdot P(F|G)$

Independence of Events:

E and F are independent If, $P(E|F) = P(E)$, ie: "whether F has occured has no effect on E"

If independent...... $P(E \cap F) = P(E) \cdot P(F)$

Mean: (Expected Value) $[E(X) = \mu$

"Tossing a Weighted Die"

X	1	2	3	4	5	6
p(X)	0.1	0.1	0.1	0.1	0.1	0.5

Expected Value = $\sum x\,p(x)$

$0.1(1+2+3+4+5) + 0.5(6)$

$= 4.5$

Discrete Random Variable

"Random variable that maps outcomes is a sample space. Numerical values are given to events in the sample space"

$$X = \begin{cases} 0 & - \text{Tails} \\ 1 & - \text{Heads} \end{cases} \qquad x = \{0, 1\}$$

Variance: Var(X)

① $Var(X) = \sum (x - E(X))^2 p(x)$

② $Var(X) = E(X^2) - [E(X)]^2$

Standard Deviation: SD(X)

$SD(X) = \sqrt{Var(X)}$

Binomial Random Variable:

① Trial results in Success/Failure

② $X \sim Bin(n, P)$

③ Trials are independent of one another

$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ [just a Histogram]

Moment Generating Function: $M_X(t)$

$M_X(t) = E(e^{tX})$

-Characterizes the distribution of a random variable X

Poisson Process:

$X \sim Poisson(\lambda t)$

Continuous Random Variable

-Takes on continuous Values

$X = [0, 100]$

Probability Density function: (pdf)

$f(x) \geq 0$ for all x

$\int_{-\infty}^{\infty} f(x)dx = 1$ [Area under graph]

$f(x) = F'(x)$

Cumulative Distribution Function: (cdf)

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$ for all x

$F(x) = \int_{-\infty}^{x} \underbrace{f(t)}_{pdf}dt$

Expected Value: E(X)

$E(X) = \int_{-\infty}^{\infty} x f(x) dx$

(1) $E(X+Y) = E(X) + E(Y)$

(2) $E(5X) = 5E(X)$

pdf, cdf Relationship

What is $P(X_1 \leq X \leq X_2)$?

$P = \int_{X_1}^{X_2} f(x)dx$

$P = F_X(X_2) - F_X(X_1)$

Variance: $Var(X) = \sigma^2$

$Var(X) = E(X^2) - [E(X)]^2$

Ex: $Var(5X + 27) = 25 Var(X)$

$Var(aX + b) = a^2 Var(X)$

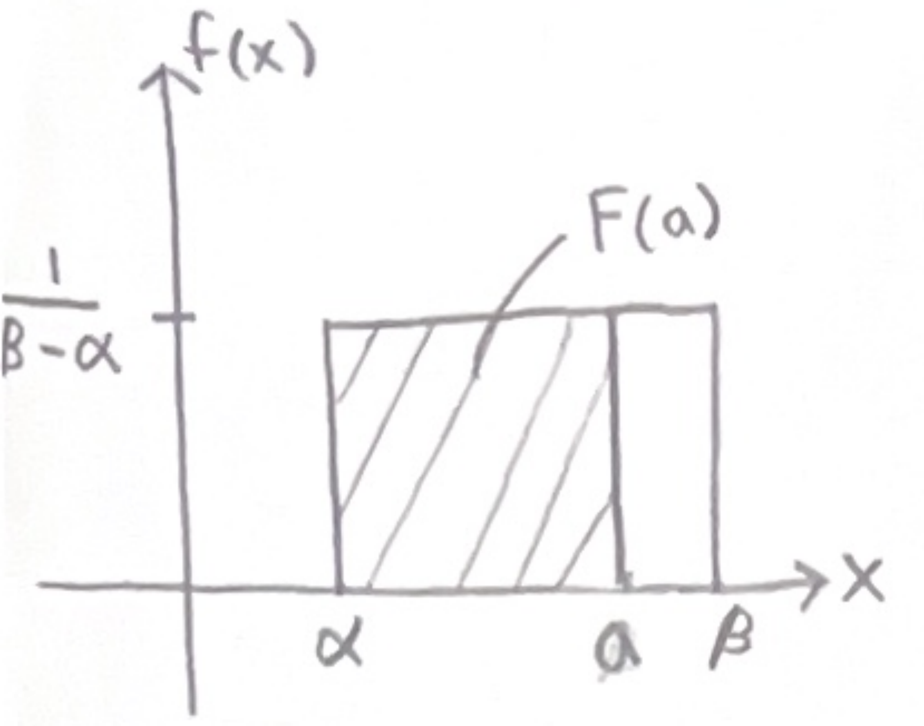
Common Continuous Distributions

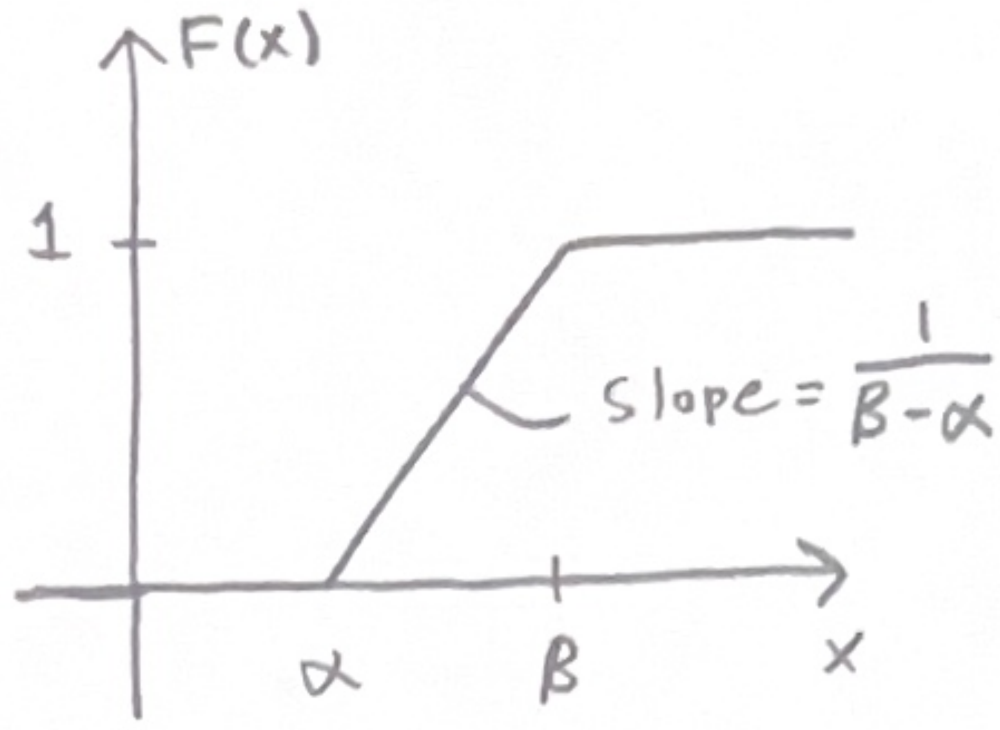
Remember: $f(x) = F'(x)$

$f(x) = pdf(X)$

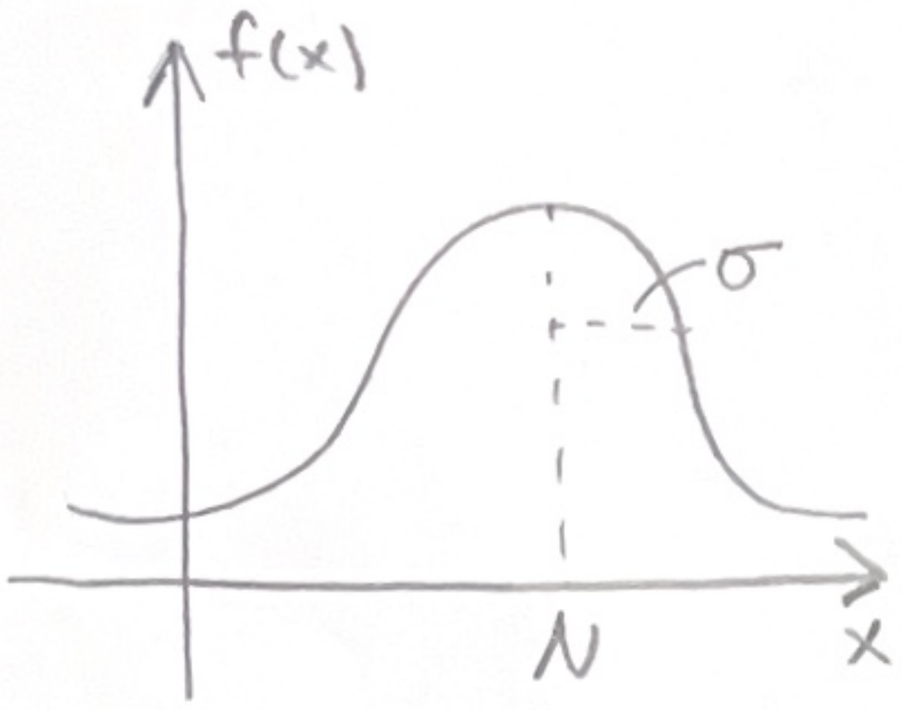
$F(x) = cdf(X)$

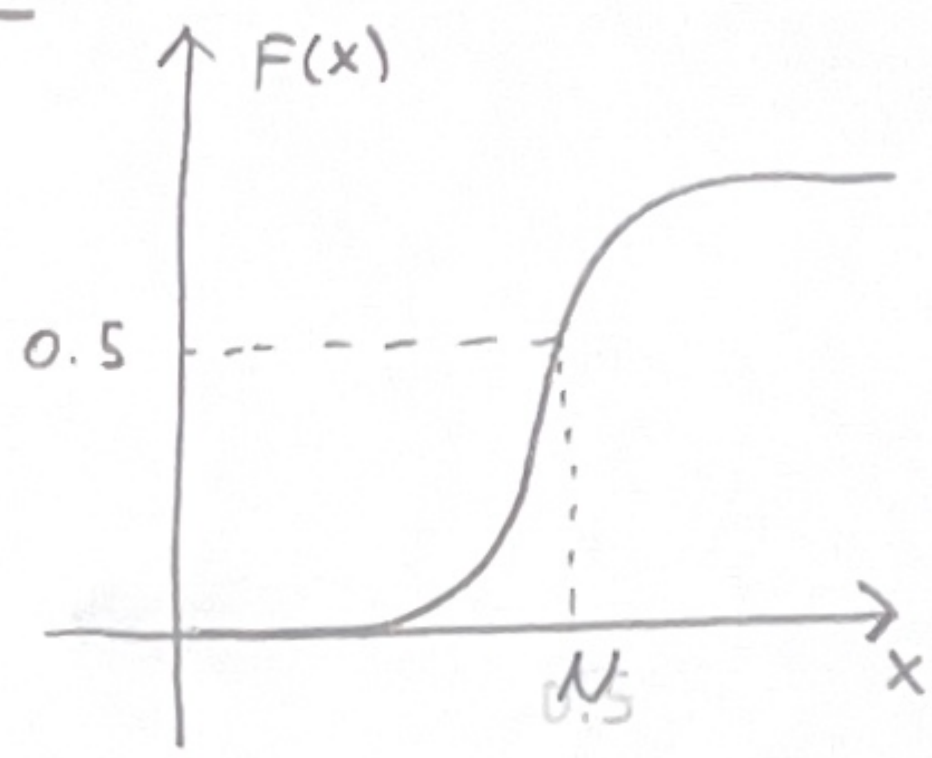
Uniform Distribution: $X \sim U(\alpha, \beta)$



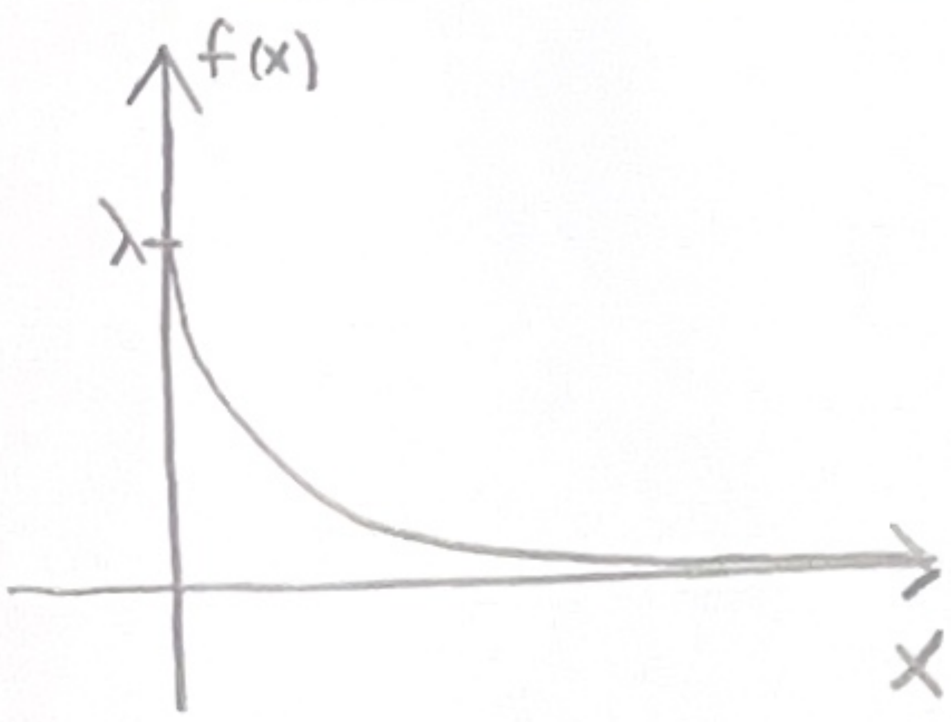


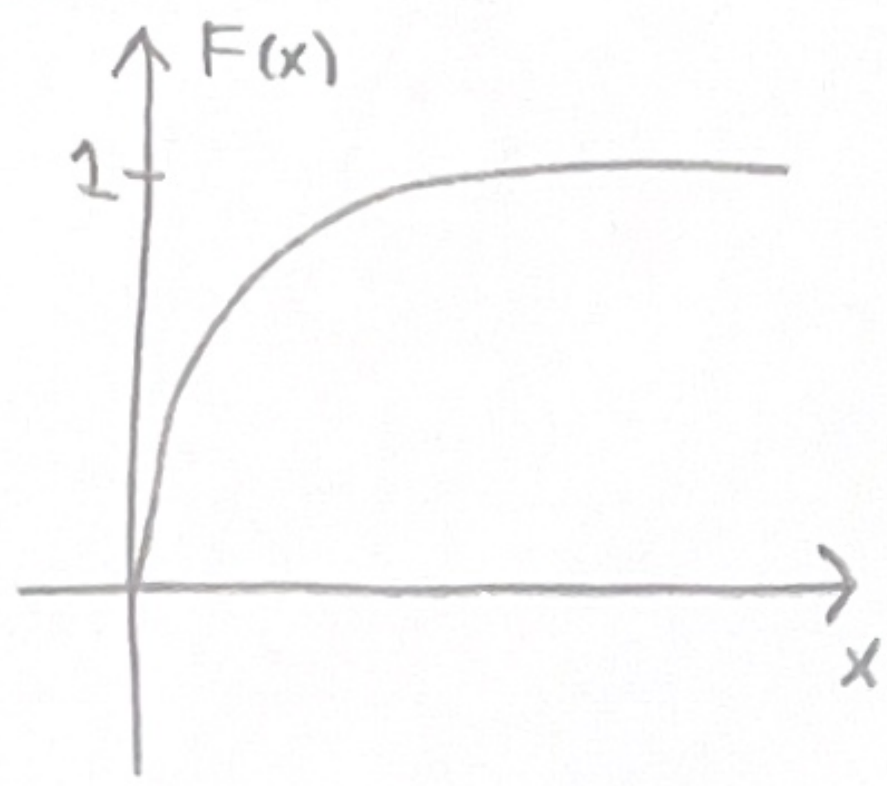
Normal Distribution: $X \sim N(\mu, \sigma^2)$





Exponential Distribution: $X \sim Exp(\lambda)$





Jointly distributed random variables

probability density function: $pdf(X,Y)$

- Continuous random variables X, Y

$$P(X \in A, Y \in B) = \iint_{A\,B} f(x,y)\,dy\,dx$$

Cumulative distribution function: cdf

$$F_{XY}(x,y) = P(X \leq x, Y \leq y)$$

$$= \int_{-\infty}^{y}\int_{-\infty}^{x} f(\alpha, \beta)\,d\alpha\,d\beta$$

Marginal PDF's:

For X: $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

For Y: $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

Expectations of functions:

$$E(g(X,Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y)\,dx\,dy$$

Covariance:

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

properties:

1. $Cov(X,X) = Var(X)$
2. $Cov(aX, bY) = ab\,Cov(X,Y)$

Correlation:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

Central limit thm: (CLT)

- The distribution Z converges to $N(0,1)$ as $n \to \infty$

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \quad \text{where} \quad \bar{X} = \frac{X_1 + X_2 \ldots X_n}{n}$$

Failure Rate:

$$\lambda(x) = \lim_{\delta \to 0} \frac{P(X \leq x + \delta \mid X > x)}{\delta}$$